

Summary of the Dissertation with Title: Algorithmic Computation of Formal Fourier Series

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Fourier series play an important role in the technical sciences and applications. The real and complex Fourier series of an integrable function $f : [a, b] \rightarrow \mathbb{R}$ are the expressions

$$\mathcal{F}(f)(t) := \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega t}, \quad (1)$$

where $\omega = \frac{2\pi}{b-a}$ is the circular frequency and the corresponding real Fourier coefficients are given by

$$a_n = \frac{2}{b-a} \int_a^b f(t) \cos(n\omega t) dt \in \mathbb{R}, \quad (2)$$

$$b_n = \frac{2}{b-a} \int_a^b f(t) \sin(n\omega t) dt \in \mathbb{R}, \quad (3)$$

whereas the complex Fourier coefficients are defined as

$$c_n = \frac{1}{b-a} \int_a^b f(t) e^{-in\omega t} dt \in \mathbb{C}. \quad (4)$$

Despite an increasing demand of those series, the method used until now to compute them via computer algebra systems (CAS) is essentially based on the same principle as in Fourier's time, i.e. using the definitions (2)–(4). Unfortunately this technique is not successful for many functions. Although numeric values of the Fourier coefficients might be available, symbolic values are often not accessible.

Modern CAS like *Maple* or *Mathematica* can compute such integrals in many cases for a given $n \in \mathbb{Z}$. However if one is interested in the Fourier coefficients for all $n \in \mathbb{Z}$, then n is considered as a given symbolic variable and such integrals can be computed only in few cases.

In this PhD. thesis, we give an algorithmic approach for the computation in symbolic form of the Fourier coefficients of a rich family of functions, namely the trigonometric holonomic functions (TH).

To do so, we first define and characterize the family of functions which can be taken as input in our algorithm, namely the set TH. For a given trigonometric holonomic function f , We first determine the trigonometric holonomic differential equation (THDE) it satisfies, then we convert this THDE into a recurrence equation (RE) for the complex Fourier coefficients of f and finally we solve that RE using the Petkovšek-van Hoeij-algorithm to get a solution in closed form. A *Maple* implementation of that algorithm is attached.